Subject specific seminars: Mathematics
What to expect:

• A brief overview of the new IB mathematics courses – the development process, the rationale for the changes, key distinctions

• Questions and answers throughout
<table>
<thead>
<tr>
<th>Year</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012/14</td>
<td>Two years of research and evaluation. Invitations issued to participants in the external review process. Research and evaluation report compiled by the Curriculum Manager. Questionnaire to schools mid 2014</td>
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<tr>
<td>2014/15</td>
<td>Circulation of Curriculum Manager’s report to review participants. First external review meeting November 2014. Internal review committee meetings in October and November 2014</td>
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<tr>
<td>2019</td>
<td>New guides, teacher support materials, videos and specimen papers to be published on the PRC February 2019 with subject specific seminars (SSS). First teaching begins in August.</td>
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<tr>
<td>2021</td>
<td>First examination of the new subjects (November 2020 last examination of the old subjects)</td>
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What’s changing?

Currently

Four mathematics subjects are offered.

➢ Further mathematics HL
➢ Mathematics HL
➢ Mathematics SL
➢ Mathematical studies SL

Final assessment of these subjects will take place in November 2020.

New

First teaching August 2019, first assessment in May 2021.

Two subjects each offered at HL and SL will increase accessibility to more students, appeal to their interests and cater for their future needs.

➢ Mathematics: analysis and approaches (HL and SL)
➢ Mathematics: applications and interpretation (HL and SL)
The NEW DP mathematics subjects

Mathematics: analysis and approaches (HL and SL)

This course recognizes the need for analytical expertise in a world where innovation is increasingly dependent on a deep understanding of mathematics. This course includes topics that are both traditionally part of a pre-university mathematics course (for example, functions, trigonometry, calculus) as well as topics that are amenable to investigation, conjecture and proof, for instance the study of sequences and series at both SL and HL, and proof by induction at HL.

The course allows the use of technology, as fluency in relevant mathematical software and hand-held technology is important regardless of choice of course. However, Mathematics: analysis and approaches has a strong emphasis on the ability to construct, communicate and justify correct mathematical arguments.
Mathematics: applications and interpretation (HL and SL)

This course recognizes the increasing role that mathematics and technology play in a diverse range of fields in a data-rich world. As such, it emphasizes the meaning of mathematics in context by focusing on topics that are often used as applications or in mathematical modelling. To give this understanding a firm base, this course also includes topics that are traditionally part of a pre-university mathematics course such as calculus and statistics.

The course makes extensive use of technology to allow students to explore and construct mathematical models. Mathematics: applications and interpretation will develop mathematical thinking, often in the context of a practical problem and using technology to justify conjectures.
Rationale for the changes

• Greater choice for students
• Changing needs of the world of work and universities
• Increasing emphasis on the use of technology
• Low uptake of Mathematics HL
• Alignment and parity of mathematics within the DP
• Very low uptake of Further mathematics HL
• Perception issues with Mathematical Studies SL
• To offer schools flexibility in the way they schedule classes
• To offer teachers flexibility in the way they teach the content
Syllabus model
The guides

Analysis

Applications

### Syllabus outline

<table>
<thead>
<tr>
<th>Syllabus component</th>
<th>Suggested teaching hours</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SL</td>
</tr>
<tr>
<td>Topic 1—Number and algebra</td>
<td>19</td>
</tr>
<tr>
<td>Topic 2—Functions</td>
<td>21</td>
</tr>
<tr>
<td>Topic 3—Geometry and trigonometry</td>
<td>25</td>
</tr>
<tr>
<td>Topic 4—Statistics and probability</td>
<td>27</td>
</tr>
<tr>
<td>Topic 5—Calculus</td>
<td>28</td>
</tr>
<tr>
<td>The toolkit and the mathematical exploration</td>
<td>30</td>
</tr>
<tr>
<td>Total teaching hours</td>
<td>150</td>
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</tbody>
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### Syllabus outline

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<th>Syllabus component</th>
<th>Suggested teaching hours—SL</th>
<th>Suggested teaching hours—HL</th>
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<tbody>
<tr>
<td>Topic 1—Number and algebra</td>
<td>16</td>
<td>29</td>
</tr>
<tr>
<td>Topic 2—Functions</td>
<td>21</td>
<td>42</td>
</tr>
<tr>
<td>Topic 3—Geometry and trigonometry</td>
<td>18</td>
<td>46</td>
</tr>
<tr>
<td>Topic 4—Statistics and probability</td>
<td>36</td>
<td>52</td>
</tr>
<tr>
<td>Topic 5—Calculus</td>
<td>10</td>
<td>41</td>
</tr>
<tr>
<td>The “toolkit” and Mathematical exploration</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Investigative, problem-solving and modelling skills development leading to an individual exploration. The exploration is a piece of written work that involves investigating an area of mathematics.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total teaching hours</td>
<td>150</td>
<td>240</td>
</tr>
</tbody>
</table>
The curriculum model at a glance

- Applications SL
  - Number and algebra: 16
  - Functions: 31
  - Geometry and trigonometry: 18
  - Statistics and probability: 19
  - Calculus: 19

- Applications HL
  - Number and algebra: 36
  - Functions: 42
  - Geometry and trigonometry: 29
  - Statistics and probability: 52
  - Calculus: 41

- Analysis SL
  - Number and algebra: 39
  - Functions: 21
  - Geometry and trigonometry: 25
  - Statistics and probability: 27
  - Calculus: 28

- Analysis HL
  - Number and algebra: 51
  - Functions: 32
  - Geometry and trigonometry: 33
  - Statistics and probability: 55
  - Calculus: 55
Key distinctions of the new subjects

1) Each subject will be available at SL and HL, with the SL course being a complete subset of the HL course.

2) There will be approximately 60 hours allocated to common SL material across both subjects.

3) 30 hours will be allocated to the development of investigational and problem solving skills, collaboration, modelling skills, and completion of the internal assessment (IA) component. This is called the “toolkit”.

4) The IA is an independent exploration of an area of mathematics chosen by the student. It is internally assessed by the teacher and externally moderated by the IB, contributing 20% to the overall level.

5) HL 3 Paper will be a 1 hour problem-solving/sustained reasoning paper – two scaffolded problems, beginning with a syllabus item and building to either a generalization or an interpretation of the problem.
The guides
Publication date Tuesday 5th February 2019

- Conceptual understanding – 12 concepts linked to MYP related concepts
- Mathematical inquiry
- Mathematical modelling
- Proof – Mathematics: analysis and approaches only
- Use of technology
- The toolkit
- Aims
- Assessment objectives
- Prior learning topics (same for both subjects)
- Syllabus sections
- Assessment sections
- Command terms and notation list
Introduction

Aims

The aims of all DP mathematics courses are to enable students to:

1. develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate mathematics clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities
7. appreciate how developments in technology and mathematics influence each other
8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course
11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics.
Assessment objectives

Problem solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP mathematics course, students will be expected to demonstrate the following:

1. **Knowledge and understanding**: Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.

2. **Problem solving**: Recall, select and use their knowledge of mathematical skills, results and models in both abstract and real-world contexts to solve problems.

3. **Communication and interpretation**: Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation; use appropriate notation and terminology.

4. **Technology**: Use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.

5. **Reasoning**: Construct mathematical arguments through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions.

6. **Inquiry approaches**: Investigate unfamiliar situations, both abstract and from the real world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.
Topic 1: Number and algebra

Concepts

Essential understandings:
Number and algebra allow us to represent patterns, show equivalencies and make generalizations which enable us to model real-world situations. Algebra is an abstraction of numerical concepts and employs variables which allow us to solve mathematical problems.

Suggested concepts embedded in this topic:
Generalization, representation, modelling, equivalence, patterns, quantity
AHL: Validity, systems.

Content-specific conceptual understandings:
- Modelling real-life situations with the structure of arithmetic and geometric sequences and series allows for prediction, analysis and interpretation.
- Different representations of numbers enable equivalent quantities to be compared and used in calculations with ease to an appropriate degree of accuracy.
- Numbers and formulae can appear in different, but equivalent, forms, or representations, which can help us to establish identities.
- Formulae are a generalization made on the basis of specific examples, which can then be extended to new examples.
- Logarithm laws provide the means to find inverses of exponential functions which model real-life situations.
- Patterns in numbers inform the development of algebraic tools that can be applied to find unknowns.
- The binomial theorem is a generalization which provides an efficient method for expanding binomial expressions.

AHL
- Proof serves to validate mathematical formulae and the equivalence of identities.
- Representing partial fractions and complex numbers in different forms allows us to easily carry out seemingly difficult calculations.
- The solution for systems of equations can be carried out by a variety of equivalent algebraic and graphical methods.
The guides

SL 4.1

<table>
<thead>
<tr>
<th>Content</th>
<th>Guidance, clarification and syllabus links</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concepts of population, sample, random sample, discrete and continuous data.</td>
<td>This is designed to cover the key questions that students should ask when they see a data set/analysis.</td>
</tr>
<tr>
<td>Reliability of data sources and bias in sampling.</td>
<td>Dealing with missing data, errors in the recording of data.</td>
</tr>
<tr>
<td>Interpretation of outliers.</td>
<td>Outlier is defined as a data item which is more than 1.5 $\times$ interquartile range (IQR) from the nearest quartile. Awareness that, in context, some outliers are a valid part of the sample but some outlying data items may be an error in the sample. Link to: box and whisker diagrams (SL4.2) and measures of dispersion (SL4.3).</td>
</tr>
<tr>
<td>Sampling techniques and their effectiveness.</td>
<td>Simple random, convenience, systematic, quota and stratified sampling methods.</td>
</tr>
</tbody>
</table>

**Connections**

**Other contexts:** Growth of bacteria or traffic to websites/social media; exponential graphs that show alarming absolute figures, but reasonable rates of growth.

**Links to other subjects:** pH semi-log curves and finding activation energy from experimental data (chemistry); exponential decay (physics); experimental work (sciences).

**TOK:** Does the applicability of knowledge vary across the different areas of knowledge? What would the implications be if the value of all knowledge was measured solely in terms of its applicability?

**Links to websites:** Gapminder makes use of log-log graphs: www.gapminder.org

Download connections template
Assessment model

SL
- Paper 1
  - 80 marks
  - 90 mins

SL
- Paper 2
  - 80 marks
  - 90 mins

HL
- Paper 1
  - 110 marks
  - 120 mins

HL
- Paper 2
  - 110 marks
  - 120 mins

HL Paper 3
- 55 marks
- 60 minutes
- Problem solving
HL Paper 3

• 20% of the overall HL grade, 55 marks, 1 hour
• 2 compulsory extended-response problem-solving questions
• GDC required
• First part of questions begin with a syllabus item
• Carefully scaffolded
• Questions require extended responses involving sustained reasoning
• Specimen papers, practice questions with examiner commentary in TSM
• Video in TSM “preparing for HL paper 3”
Teacher support materials (TSM)

The new subject websites will each include a section called “in practice”

- Structure and connections:
  - Making connections across the mathematics curriculum
  - Different models for structuring classes
- Preparing for the IA and the new HL paper 3:
  - HL paper 3 specimen and practice questions with commentary from examiners
  - Detailed guidance on the IA process and supporting students
- The toolkit - activities and guidance to support:
  - The approaches to teaching and learning in mathematics
  - Modelling
  - Use of technology
  - Some of the new content such as proof, Voronoi diagrams, financial mathematics, phase portraits
  - Cognitive academic language proficiency (CALP) in mathematics
- Videos:
  - Course selection – communicating choices
  - Technology in the mathematics classroom
  - The “toolkit”
  - Preparing for HL paper 3